Therefore, the possibility of bounded resonances depends not only on ω but also on a: as $a \to 0$ a bounded resonance in mass is possible at any frequency $\omega > \pi/2$.

The study of the dependence of the contact stiffness on the frequency is of independent interest. Such results have been obtained earlier for large rectangular stamps (see /5/ and the bibliography given there). Curves of $|P_1(\omega)|$, $\theta_1(\omega)$ are presented in Fig.2 for different values of a.

As we noted above, the fact that for small a the quantity P_1 remains constant over a broad frequency range is essentially new. As before, $P_1 = 0$ at the layer natural vibration frequencies ($\omega = 2.89$; 2.93; 7.64; 8.82) including also for double $\zeta_k \neq 0$ on the left boundary of the reverse wave ranges (the dependence $P_1(\omega)$ in the reverse wave range was given earlier in a coarse scale*). (*Babeshko V.A., Glushkov E.V. and Glushkova N.V., On the Resonance Properties of a Stamp Elastic Layer System. Unpublished Manuscript 8329-B VINITI. December 4, 1985.)

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PULSE PROPAGATION IN MEDIA WITH SMALL VELOCITY DISPERSION AND RELAXATION TIME SPECTRUM OF THE FORM $1/\tau$. EXACT SOLUTION*

S.Z. DUNIN and G.A. MAKSIMOV

Pulse propagation in a medium whose dispersion-dissipative properties correspond to the presence of relaxation mechanisms in the medium, whose relaxation times form a spectrum of the form $g(\tau) \sim 1/\tau$, is considered. In the case of small velocity dispersion an exact solution is obtained for the pulse shape and it is shown that it is equivalent to the description of pulse propagation in a medium with "*Ei*-memory".

Acoustic wave propagation in real media can often be considered within the framework of a linear approximation of hereditary elasticity theory (HET) /1/. Phenomenological HET coefficients can be obtained using the theory of internal parameters /2/ characterized by relaxation times to a thermodynamic equilibrium state.

Exact solutions are known only for certain rheological models of media: a standard body /3/ characterized by a single relaxation time, its limiting case of Voigt /4, 5/ and Maxwell /6, 7/ media; in the case of small velocity dispersion for the model of a medium with "*E*-memory

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/8, 9/. Certain solutions can be obtained using a modified HET /10/* (*See also: Nigul.U., The modified theory of viscoelasticity. Preprint, Academy of Sciences of the Estonian SSR, Tallinn, 1983.) in which modified HET kernels (MK) associated in a definite way with ordinary HET kernels are selected as the initial one. Thus, MK of exponential form correspond to a medium with "*E*-memory", and MK in the form of the difference between two integral exponents correspond to a medium with "Ei-memory" /10/; the solution is obtained for the last of these media for certain particular values of the parameters.

A number of experimental facts are not explained successfully in the description of wave propagation in geo- and biomedia within the framework of the simplest rheological models, for instance, the linear dependence of the wave damping factor on the frequency over a broad frequency range /11-15/. The following explanation exists for this fact within the framework of linear theories: both geo- and biomedia are distinguished by the complexity of their microstructure and its hierarchy consequently, relaxation mechanisms can be associated with the most diverse structural features of such media, and consequently, their relaxation times form a spectrum. By selecting the parameters of this spectrum, it can be arranged that a linear dependence of the damping factor on the frequency is obtained over a fairly broad, but bounded, frequency domain. It is shown in a number of papers /13, 14/ that the relaxation time spectrum should have the form $g(\tau) \sim \tau^{-1}$ for a satisfactory description of experimental data.

1. Small-amplitude acoustic waves are described by the equations of linear elasticity theory with an equation of state of hereditary type /11/

$$\sigma_{ij}(\mathbf{r}, t) = \int_0^t [\varepsilon_{ij}(\mathbf{r}, t') M(t-t') + \delta_{ij}\varepsilon_{ll}(\mathbf{r}, t') L(t-t')]\, dt'$$

Considering the variety of dispersion-dissipative properties of media that can be described in the terminology of exponential relaxation mechanisms, the general form of the kernels $M(t)$ and $L(t)$ can be written in the form

$$M(t) \sim L(t) \sim \rho_0 \varkappa(t)$$

$$\varkappa(t) = c_\infty^2 \left[\delta(t) - \frac{\Delta}{B}\int_a^b g(\tau) \exp\left(-\frac{t}{\tau}\right) d\tau\right]$$

$$B = \int_a^b g(\tau)\,\tau\, d\tau, \quad \Delta = \frac{c_\infty^2 - c_0^2}{c_\infty^2}$$

where Δ is the velocity dispersion, and c_∞ and c_0 are the largest and smallest possible wave propagation velocities (longitudinal or transverse) in the medium.

The relaxation time spectrum $g(\tau)$ possesses the following properties:

$$g(\tau) \geqslant 0, \quad \int_a^b g(\tau)\, d\tau = 1, \quad g(\tau = 0) = 0$$

This latter property is associated with the fact that the contribution of processes with zero relaxation time corresponding to ideal elasticity is represented separately in the form of a delta-function.

We will examine plane wave propagation. It has been shown /16/ how a solution of problems with more complex geometry can be obtained using the solution of this problem.

Green's function of the plane problem obeys the following wave equation (the Laplace transformation $(t \to p)$ is made in the time variable):

$$[\partial^2/\partial x^2 - K^2(p)]\, I(x, p) = \delta(x), \quad K^2(p) = p^2/\varkappa(p)$$

whose solution can be represented in the form of a Mellin integral

$$I(t, x) = \frac{1}{2\pi i}\int_\gamma \exp\{pt - xK(p)\}\, dp, \quad \gamma = (-i\infty, +i\infty) \tag{1.1}$$

The properties of this integral are determined by the form of the dependence

$$\varkappa(p) = c_\infty^2 \left[1 - \frac{\Delta}{B}\int_a^b \frac{g(\tau)}{p + 1/\tau}\, d\tau\right]$$

In the case of the relaxation time spectrum $g(\tau) = (\tau \ln(b/a))^{-1}$, $\tau \in [a, b]$ (the spectrum normalization conditions is taken into account), we obtain

$$K(p) = c_\infty^{-1} p \left[1 - \Delta (b-a)^{-1} P(p)\right]^{-1/2}, \quad (1.2)$$
$$P(p) = p^{-1} \ln((bp+1)/(ap+1))$$

The function $P(p)$ is analytic in the domain $\operatorname{Re} p > -1/b$ and, therefore, attains its highest value on the domain boundary namely, for $p = -1/b$. Consequently, for small velocity dispersion $\Delta \ll 1$ a domain $\operatorname{Re} p > \delta > -1/b$ exists in which the absolute value of the second component in the square brackets in (1.2) will be less than one. The expression

$$\delta = -b^{-1}\left[1 - \exp\{-\Delta^{-1}(1 - a/b)\}\right]$$

can be obtained to estimate the quantity δ, from which it follows that $\delta \in (-1/b, 0]$. In particular, expansion in the small parameter $\Delta \ll 1$ is possible in the domain $\operatorname{Re} p \geqslant 0$. Taking account of the first two terms in this expansion, we go over to evaluation of the integral (1.1).

$$I(t, x) = \frac{1}{2\pi i} \int_\gamma \left(\frac{bp+1}{ap+1}\right)^{-\Omega} \exp(pt')\, dp, \quad t' = t - \frac{x}{c_\infty}, \quad \Omega = \frac{\Delta x}{2(b-a)c_\infty} \quad (1.3)$$

Utilizing the Efros theorem on a generalized convolution /17/ this integral can be transformed to the form

$$I(t, x) = \int_0^\infty F(\Omega, \zeta) Q(t', \zeta)\, d\zeta, \quad F(\Omega, \zeta) = \frac{\zeta^{\Omega-1}}{\Gamma(\Omega)} \quad (1.4)$$
$$Q(t', \zeta) = \exp\left\{-\frac{t'}{a} - \zeta \frac{b}{a}\right\} \frac{\partial}{\partial t'} \left\{\Theta(t') I_0\left(2\sqrt{\frac{b-a}{a^2} t'\zeta}\right)\right\}$$

Here $\Gamma(z)$ is the Gamma function, $I_0(z)$ is the Bessel function of imaginary argument, and $\Theta(z)$ is the Heaviside unit function. Evaluating the integral (1.4) /18, 19/, we obtain

$$I(t, x) = \left(\frac{a}{b}\right)^\Omega \exp\left(-\frac{t'}{a}\right) \frac{\partial}{\partial t'} \{\Theta(t')\, {}_1F_1(\Omega, 1, \lambda t')\}, \quad \lambda = \frac{b-a}{ab} \quad (1.5)$$

where ${}_1F_1(\alpha, \beta, z)$ is the degenerate hypergeometric function of the first kind (the Kummer function).

Expression (1.3) is formally identical with that which has been obtained for a pulse propagating in a medium with "Ei-memory". For $\Omega = 1$ the solution presented in /10/ is obtained both directly from (1.3) and from (1.5) since the relationship ${}_1F_1(a, a, z) = e^z$ /20/ holds, say.

2. Note that in the limit as $b \to a \equiv \tau$ expression (1.5) is identical with the expression corresponding to the solution for a medium with a single relaxation time τ (i.e., a relaxation time spectrum of the form $g(\tau') = \delta(\tau' - \tau)$). Indeed $\Omega \to \infty$ as $b \to a$. Consequently, by using the limit relationship for the degenerate hypergeometric function /20/, we obtain the expression

$$I(t, x) = \exp\left\{-\frac{t'}{\tau} - \Omega'\right\} \frac{\partial}{\partial t'} \left\{\Theta(t') I_0\left(2\sqrt{\frac{\Omega' t'}{\tau}}\right)\right\} \quad (2.1)$$
$$\Omega' = \frac{\Delta x}{2c_\infty \tau} = \lim_{b \to a \equiv \tau} \left\{\Omega\left(1 - \frac{a}{b}\right)\right\}$$

that agrees in accuracy with the result in /9/.

We will examine other limit cases resulting from representation (1.5) which we rewrite for convenience, in the form

$$I(t, x) = \left(\frac{a}{b}\right)^\Omega \delta(t') + \Omega\lambda \left(\frac{a}{b}\right)^\Omega \exp\left(-\frac{t'}{a}\right) {}_1F_1(\Omega + 1, 2, \lambda t')\, \Theta(t') \quad (2.2)$$

The properties of the Kummer function /20/ were taken into account here.

It follows from relationship (2.1) that a pulse front exists that moves with velocity c_∞ (i.e., $I(t, x) \equiv 0$ for $t < x/c_\infty$). The first component in (2.2) describes the predecessor which propagates together with the front, that damps out exponentially with distance as

$\exp\{-\Omega \ln (b/a)\}$. The principal part of the pulse (its body) is described by the second term in (2.2).

We will use the representation of the function ${}_1F_1$ in the different limiting cases /20/ to clarify the nature of the behaviour of the body of the pulse. Utilizing the expansion of the function ${}_1F_1(\alpha, \beta, z)$ for small values of z, it can be seen that in the neighbourhood of the front $\frac{1}{2}(\Omega+1)t'\lambda \ll 1$ the pulse amplitude increases with distance from the front if the pulse has traversed a sufficiently large distance $\Omega > (b+a)/(b-a)$, and decreases otherwise. We note that the same pulse behaviour is also observed near the front for a medium with a single relaxation time but at distances $\Omega' \gtrless 1/2$.

By using the expansion of the function ${}_1F_1(\alpha, \beta, z)$ for large values of z we obtain a profile of the body of the pulse for large times $\lambda t' \gg 1$ (the asymptotic representation of the Gamma function is also used)

$$I(t, x) = (2\pi\Omega)^{-1/2} \frac{\Omega}{t'} \exp\left\{-\frac{t'}{b} + \Omega \ln\left(\frac{t'}{\Omega} \frac{b-a}{b^2} e\right)\right\} \tag{2.3}$$

For $b > a$ the exponent in this expression is always less than zero. Therefore, relationship (2.3) describes the exponentially damped "tail" part of the pulse.

At sufficiently large distances from the source $\Omega \gg 1$ for time intervals satisfying the condition $1 < \lambda t' < \Omega^{1/3}$, such that the following asymptotic form /20/

$$ {}_1F_1(\alpha, \beta, z) = \Gamma(\beta) e^{z/2} (z(\beta/2 - \alpha))^{1/2-\beta/2} J_{\beta-1}(2\sqrt{z(\beta/2-\alpha)})$$
$$\alpha \gg 1, \ |z| = |\beta/2 - \alpha|^{\rho}, \ 0 \leqslant \rho < 1/3$$

is valid for ${}_1F_1(\alpha, \beta, z)$, the representation

$$I(t, x) = \left(\frac{a}{b}\right)^{\Omega} \exp\left\{-\frac{\mu t'}{2}\right\} \left(\frac{\lambda\Omega}{t'}\right)^{1/2} I_1(2\sqrt{\lambda\Omega t'}), \quad \mu = \frac{b+a}{ab} \tag{2.4}$$

can be obtained that is identical with the representation (2.1) in the limiting case as $b \to a$. The expression (2.4) provides no description of the most interesting domain where the pulse body has a maximum. This can be seen if the asymptotic representation of the Bessel function of imaginary argument is used and the value of the exponential is determined at a point where it reaches a maximum. Namely, for $t' = 4\Omega\lambda/\mu^2$ the expression in the exponential is less than zero is all cases except $b = a$.

To obtain an expression describing the neighbourhood of the maximum of the pulse body we use representation (1.4). The contribution from the neighbourhood of the lower limit of integration can be neglected at large distances $\Omega > 1$ because of the second formula in (1.4); then its asymptotic representation can be used instead of the Bessel function of imaginary argument in the last relationship of (1.4) under the condition $b/a\,[\lambda t'/(\Omega-1)]^{1/2} \gg 1$. Integrating with respect to ζ in (1.4) after this /21/, we obtain for the pulse body (the predecessor is the same as in (2.2)):

$$I(t, x) = \frac{1}{\sqrt{\pi}} \left(\frac{a}{2b}\right)^{\Omega} \frac{\Gamma(2\Omega + 1/2)}{\Gamma(\Omega)} \left(\frac{\lambda}{2t'}\right)^{1/4} \exp\left\{-\frac{\mu t'}{2}\right\} D_{-2\Omega-1/2}(-\sqrt{2\lambda t'})$$

where $D_\nu(z)$ is a parabolic cylinder function ($D_{-a-1/2}(x) \equiv U(a, x)$ is a Whittaker function). Using the asymptotic forms of the Gamma function for $\Omega \gg 1$, as well as the Darwin expansion for the Whittaker function /20/ ($a > 0$, $x^2 + 4a \gg 1$), we obtain after reduction

$$\begin{aligned} I(t, x) = {} & (2\pi t')^{-1/2} \Lambda^{-1/4} \exp\{-{}^1/_2\mu t' + {}^1/_2\lambda t' \Lambda^{1/2} - \\ & \Omega \ln(b/a) + \Omega \ln[1 + {}^1/_2(\lambda t'/\Omega)(1 + \Lambda^{1/2})]\}, \\ & \Lambda = 1 + 4\Omega/(\lambda t') \end{aligned} \tag{2.5}$$

The expression under the exponential sign in (2.5) has a zero maximum for $t' = \Omega(b-a)$ as will be shown below. Therefore, relationship (2.5) describes the pulse body in the neighbourhood of its maximum. The amplitude of the latter falls off as the traversed distance increases occurs according to the power law

$$(2\pi\Omega(b^2 - a^2))^{-1/2} \equiv (\pi\Delta x(b+a)/c_\infty)^{-1/2}$$

3. A number of the results obtained above for the analysis of the exact solution (1.5)

can be obtained by a simpler method by using approximate methods directly to evaluate the integral (1.1). For instance, the structure $I(t, x)$ in the neighbourhood of the front is determined by expanding $K(p)$ as $p \to \infty$ and has the form /22, 23/

$$I(t, x) = (a/b)^{\Omega}[\delta(t') + (A\Omega/t')^{1/2} I_1(2(A\Omega t')^{1/2})\,\theta(t')] \tag{3.1}$$
$$A = \lambda - {}^3/_4\Delta(b-a)^{-1}\ln^2(b/a)$$

Expression (3.1) in the case when $\Delta \ll 1$ is in complete agreement with (2.4) and (2.2).

At large distances from the source $(\Omega \gg 1)$ an approximate expression for the pulse profile can be obtained by using the saddle-point method:

$$I(t, x) = (2\pi\Omega S_2(p_n))^{-1/2}\exp\{\Omega S(p_n)\}. \tag{3.2}$$
$$S(p_n) = -\frac{p_n(b-a)}{(p_n b+1)(p_n a+1)} - \ln\frac{bp_n+1}{ap_n+1}, \quad S_2(p_n) = \frac{(b-a)(2abp_n+b+a)}{(p_n b+1)^2(p_n a+1)^2}$$
$$p_n = -\mu/2 + \lambda\Lambda^{1/2}/2$$

Expression (3.2) is completely equivalent to (2.5)'. This can be seen by expressing all the parameters in terms of the ratio Ω/t' and carrying out the necessary reduction.

It follows from the last formula in (3.2) that the correspondence between the saddle point $p_n > -1/b$ and the parameter Ω/t' is one-to-one. Consequently, the maximum of the pulse body can be determined from the condition $S'(p_n) = 0$, from which it follows that $p_n = 0$. This corresponds to the value $t' = \Omega(b-a)$.

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ON THE PLASTIC LOADING PROCESS BEHIND AN UNLOADING SHOCK FRONT*

A.G. BYKOVTSEV

The problem of elastic wave refraction in an elastic-plastic half-space (EPH) in the active loading domain has been investigated /1-4/ for different models of an elastic-plastic body. The problem has been solved /5/ for refraction of a pure shear elastic wave that has a profile of steps of finite length in an EPH in both the active plastic loading domain and in the unloading zone under the assumption that the material behind the unloading shock (US) is in the elastic state. It is shown below for this problem that a plastic loading process can be realized behind the US front and a solution is constructed in the secondary plastic flow zone.

1. A medium is under antiplane deformation conditions when a pure shear wave propagates. The displacement velocity vector w is directed along the x_3 axis and depends on the variables x_1, x_2 and the time t, and the stresses $\tau_1 = \sigma_{13}(x_1, x_2, t)$, $\tau_2 = \sigma_{23}(x_1, x_2, t)$ differ from zero. Henceforth we will confine ourselves to investigating selfsimilar solutions of the equations of the dynamics of an ideal elastic-plastic body that depend on two variables $x = x_1 - ct$ and $y = x_2$. In this case the equations of the characteristics and the relationships along the characteristics of the system of motion equations have the following form /3/:

in the elastic domain and unloading zone

$$x + \varkappa y = \text{const}, \quad \varkappa w - \tau_2 = \text{const} \tag{1.1}$$

$$x - \varkappa y = \text{const}, \quad \varkappa w + \tau_2 = \text{const} \tag{1.2}$$

$$y = \text{const}, \quad \tau_1 + w = f(y), \quad \varkappa = \sqrt{M^2 - 1}, \quad M = c/a, \quad a = \sqrt{\mu/\rho} \tag{1.3}$$

in the active plastic loading domain

$$dy\,(M + \cos\theta) = -\sin\theta\, dx, \quad \theta + Mw = \text{const} \tag{1.4}$$

$$dy\,(M - \cos\theta) = \sin\theta\, dx, \quad \theta - Mw = \text{const}, \quad \tau_1 = \sin\theta, \quad \tau_2 = \cos\theta \tag{1.5}$$

Here ρ is the density and μ is the shear modulus.

Equations (1.1)-(1.5) are written in dimensionless variables that will be used later (to simplify the writing the bars above the dimensionless variables are omitted, and k is the yield point)

$$\bar{x} = \frac{x}{l}, \quad \bar{y} = \frac{y}{l}, \quad \bar{\tau}_1 = \frac{\tau_1}{k}, \quad \bar{\tau}_2 = \frac{\tau_2}{k}, \quad \bar{w} = \frac{w}{w^*}, \quad w^* = \frac{ck}{\mu}$$

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